FORÇAS DE COULOMB E CAMPO ELÉCTRICO
COULOMB FORCES AND ELECTRIC FIELD

1) Resolução: Resolution:

Q1 = 0,5mC; (0,0,0)m=P1, ponto 1, point 1

Q2 = 0,1mC; (2,1,2)m=P2, ponto 2, point 2 $\varepsilon = \varepsilon_0$ $\vec{E} = \frac{Q}{4\pi\varepsilon R^2}\vec{a}_R$ $\vec{F} = Q\vec{E}$

a) Determine o campo eléctrico no local de Q1, na sua ausência. Calculate the electric field at the location of Q1 in its absence.

$$\vec{E}_{21} = \frac{Q_2}{4\pi\varepsilon R^2}\vec{a}_R \qquad \vec{E}_{21} = \frac{Q_2}{4\pi\varepsilon|P_1 - P_2|^2}\frac{P_1 - P_2}{|P_1 - P_2|}$$

$\vec{E}_{21}$ Campo eléctrico provocado por Q2 no ponto 1; Electrical field generated by Q2 in the point 1

$$\vec{E}_{21} = \frac{0.1\times10^{-3}\,\text{C}}{4\pi 8.854\times10^{-12}\,\text{F/m}\left|(0\vec{u}_x + 0\vec{u}_y + 0\vec{u}_z)\,\text{m} - (2\vec{u}_x + 1\vec{u}_y + 2\vec{u}_z)\,\text{m}\right|^2}\frac{(0\vec{u}_x + 0\vec{u}_y + 0\vec{u}_z)\,\text{m} - (2\vec{u}_x + 1\vec{u}_y + 2\vec{u}_z)\,\text{m}}{\left|(0\vec{u}_x + 0\vec{u}_y + 0\vec{u}_z)\,\text{m} - (2\vec{u}_x + 1\vec{u}_y + 2\vec{u}_z)\,\text{m}\right|}$$

$$\vec{E}_{21} = \frac{0.1\times10^{-3}}{4\pi 8.854\times10^{-12}\left|-2\vec{u}_x - 1\vec{u}_y - 2\vec{u}_z\right|^2}\frac{-2\vec{u}_x - 1\vec{u}_y - 2\vec{u}_z}{\left|-2\vec{u}_x - 1\vec{u}_y - 2\vec{u}_z\right|}\,\text{C/F/m}$$

$$\vec{E}_{21} = \frac{0.1\times10^{-3}}{4\pi 8.854\times10^{-12}\times 9}(-2\vec{u}_x/3 - \vec{u}_y/3 - 2\vec{u}_z/3)\,\text{C/F/m}$$

$$\vec{E}_{21} = 99.864\times10^3(-2\vec{u}_x/3 - \vec{u}_y/3 - 2\vec{u}_z/3)\,\text{C/F/m}, \qquad \text{F = A.s/V}, \qquad \text{C = A.s}$$

$$\vec{E}_{21} = 99.864\times10^3(-2\vec{u}_x/3 - \vec{u}_y/3 - 2\vec{u}_z/3)(\text{A}\cdot\text{s})/(\text{A}\cdot\text{s/V})/\text{m}$$

$$\vec{E}_{21} = 99.864\times10^3(-2\vec{u}_x/3 - \vec{u}_y/3 - 2\vec{u}_z/3)\,\text{V/m}$$

Resposta: Answer: $\vec{E}_{21} = -66.58\vec{u}_x - 33.294\vec{u}_y - 66.58\vec{u}_z$ (kV/m)

b) Determinar a força exercida sobre Q1. Calculate the force acting on Q1.

$\vec{F}_{21} = Q_1 \vec{E}_{21}$ $\vec{F}_{21} = 0.5\times10^{-3}\,\text{C}\times 99.864(-2\vec{u}_x/3 - \vec{u}_y/3 - 2\vec{u}_z/3)\,\text{kV/m}$

$\vec{F}_{21} = 49.932(-2\vec{u}_x/3 - \vec{u}_y/3 - 2\vec{u}_z/3)\,\text{J/m}$ $\vec{F}_{21} = 49.932(-2\vec{u}_x/3 - \vec{u}_y/3 - 2\vec{u}_z/3)\,\text{N}$

Resposta: Answer: $\vec{F}_{21} = -33.29\vec{u}_x - 16.64\vec{u}_y - 33.29\vec{u}_z$ (N)

2) Resolução: Resolution:

Q3 = -0.2mC; (1,2,0)m=P3, ponto 3; point 3

a) Recalcule 1)a). Recalculate 1)a). Determine o campo eléctrico no local de Q1, na sua ausência. Calculate the electric field at the location of Q1 in its absence.

$\vec{E}_{31} = \frac{Q_3}{4\pi\varepsilon R^2}\vec{a}_R$ $\vec{E}_{31} = \frac{Q_3}{4\pi\varepsilon|P_1 - P_3|^2}\frac{P_1 - P_3}{|P_1 - P_3|}$

$\vec{E}_{31}$ Campo eléctrico provocado por Q3 no ponto 1; Electrical field generated by Q3 in the point 1

$$\vec{E}_{31} = \frac{-0.2\times10^{-3}\,\text{C}}{4\pi 8.854\times10^{-12}\,\text{F/m}\left|(0\vec{u}_x + 0\vec{u}_y + 0\vec{u}_z)\,\text{m} - (1\vec{u}_x + 2\vec{u}_y + 0\vec{u}_z)\,\text{m}\right|^2}\frac{(0\vec{u}_x + 0\vec{u}_y + 0\vec{u}_z)\,\text{m} - (1\vec{u}_x + 2\vec{u}_y + 0\vec{u}_z)\,\text{m}}{\left|(0\vec{u}_x + 0\vec{u}_y + 0\vec{u}_z)\,\text{m} - (1\vec{u}_x + 2\vec{u}_y + 0\vec{u}_z)\,\text{m}\right|}$$

$$\vec{E}_{31} = \frac{-0.2\times10^{-3}}{4\pi 8.854\times10^{-12}\left|-1\vec{u}_x - 2\vec{u}_y + 0\vec{u}_z\right|^2}\frac{-1\vec{u}_x - 2\vec{u}_y + 0\vec{u}_z}{\left|-1\vec{u}_x - 2\vec{u}_y + 0\vec{u}_z\right|}\,\text{C/F/m}$$

$\vec{E}_{31} = \frac{-0.2\times10^{-3}}{4\pi 8.854\times10^{-12}\times 5}(-\vec{u}_x/\sqrt{5} - 2\vec{u}_y/\sqrt{5})\,\text{C/F/m}$

$\vec{E}_{31} = 359.51\times10^3(\vec{u}_x/\sqrt{5} + 2\vec{u}_y/\sqrt{5})\,\text{C/F/m}$, F =A.s/V, C=A.s

$\vec{E}_{31} = 359.51\times10^3(\vec{u}_x/\sqrt{5} + 2\vec{u}_y/\sqrt{5})\,(\text{A}\cdot\text{s})/(\text{A}\cdot\text{s/V})/\text{m}$

$\vec{E}_{31} = 359.51\times10^3(\vec{u}_x/\sqrt{5} + 2\vec{u}_y/\sqrt{5})\,\text{V/m}$ $\vec{E}_{31} = 359.51\times(\vec{u}_x/\sqrt{5} + 2\vec{u}_y/\sqrt{5})\,\text{kV/m}$

$\vec{E}_1 = \vec{E}_{21} + \vec{E}_{31}$ $\vec{E}_1 = 99.864(-2\vec{u}_x/3 - \vec{u}_y/3 - 2\vec{u}_z/3)\,\text{kV/m} + 359.51\times(\vec{u}_x/\sqrt{5} + 2\vec{u}_y/\sqrt{5})\,\text{kV/m}$

$\vec{E}_1 = 99.864(-2\vec{u}_x/\sqrt{45} - \vec{u}_y/\sqrt{45} - 2\vec{u}_z/\sqrt{45})\sqrt{5} + 359.51\times(\vec{u}_x/\sqrt{45} + 2\vec{u}_y/\sqrt{45})3$

$$\vec{E}_1 = \left((-2\sqrt{5}\times 99.864 + 359.51\times 3)\vec{u}_x + (-99.864\sqrt{5} + 359.51\times 2\times 3)\vec{u}_y - 2\sqrt{5}\times 99.864\vec{u}_z\right)/\sqrt{45}$$

$$\vec{E}_1 = \left(632\vec{u}_x + 1934\vec{u}_y - 447\vec{u}_z\right)/\sqrt{45}$$

Resposta: Answer: $\vec{E}_1 = 94.2\vec{u}_x + 228.3\vec{u}_y - 66.6\vec{u}_z$ (kV/m)

b) Recalcule 1)b). Recalculate 1)b). Determinar a força exercida sobre Q1. Calculate the force acting on Q1.

$$\vec{F}_1 = Q_1\vec{E}_1$$

$$\vec{F}_1 = 0.5\times 10^{-3}\ \text{C}(94.2\vec{u}_x + 228.3\vec{u}_y - 66.6\vec{u}_z)\ \text{kV/m} \qquad \vec{F}_1 = (47.1\vec{u}_x + 114.2\vec{u}_y - 33.3\vec{u}_z)\ \text{N}$$

Resposta: Answer: $\vec{F}_1 = 47.1\vec{u}_x + 114.2\vec{u}_y - 33.3\vec{u}_z$ (N)

c) Determine a força exercida por cada uma das cargas, Q2 e Q3, sobre Q1 e calcule a sua soma vectorial. Calculate the force of each of the charges, Q2 and Q3 over Q1 and calculate their vector sum.

$$\vec{F}_{21} = Q_1\vec{E}_{21}$$
$$\vec{F}_{31} = Q_1\vec{E}_{31}$$

$$\vec{E}_{21} = -66.58\vec{u}_x - 33.294\vec{u}_y - 66.58\vec{u}_z \quad \text{(kV/m)}$$
$$\vec{E}_{31} = 359.51\times(\vec{u}_x/\sqrt{5} + 2\vec{u}_y/\sqrt{5})\ \text{kV/m}$$

$$\vec{F}_{21} = 0.5(-66.58\vec{u}_x - 33.294\vec{u}_y - 66.58\vec{u}_z)$$
$$\vec{F}_{31} = 0.5\times 359.51\times(\vec{u}_x/\sqrt{5} + 2\vec{u}_y/\sqrt{5})$$

$$\vec{F}_{21} = -33.29\vec{u}_x - 16.65\vec{u}_y - 33.29\vec{u}_z$$
$$\vec{F}_{31} = 80.39\vec{u}_x + 160.78\vec{u}_y$$

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} \qquad \vec{F}_1 = 47.1\vec{u}_x + 144.2\vec{u}_y + 33.3\vec{u}_z$$

Resposta: Answer: $\vec{F}_1 = 47.1\vec{u}_x + 114.2\vec{u}_y - 33.3\vec{u}_z$ (N)

3) $Q = 1\mu C$ $\varepsilon = \varepsilon_0$ d=10cm ρ_s=-3 $\mu C/m^2$ $\vec{E} = \frac{\rho_s}{2\varepsilon}\vec{a}_R$

a) Determine o campo eléctrico no local de Q sem a sua presença. Calculate the electric field at Q without his presence.

$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\vec{a}_R$ $\vec{E} = \frac{-3\times10^{-6}\ C/m^2}{2\times8.854\times10^{-12}\ F/m}\vec{a}_R$ $\vec{E} = -169.4\vec{a}_R\ kV/m$

Resposta: Answer: $\vec{E} = -169.4\vec{a}_N$ (kV/m)

$\vec{a}_N$: é o vector unitário perpendicular ao plano no sentido deste para o ponto. is the unit vector perpendicular to the plane and from it to de point.

b) Determine a força, modulo direcção e sentido, a que a carga Q está submetida. Calculate the strength, direction and magnitude which Q is submitted.

$\vec{F} = Q\vec{E}$ $\vec{F} = 1\times10^{-6}\ C(-169.4\times10^3\vec{a}_N)\ V/m$ $\vec{F} = -169.4\vec{a}_N\ mN$

Resposta: Answer: $\vec{F} = -169.4\vec{a}_N\ mN$

4) $Q = 1\mu C$ $\varepsilon = \varepsilon_0$ d=10cm ρ_l=-3 $\mu C/m$ $\vec{E} = \frac{\rho_l}{2\pi\varepsilon R}\vec{a}_R$

a) Determine o campo eléctrico no local de Q sem a sua presença. Calculate the electric field at Q without his presence.

$\vec{E} = \frac{\rho_l}{2\pi\varepsilon R}\vec{a}_R$ $\vec{E} = \frac{-3\times10^{-6}\ C/m}{2\pi\times8.854\times10^{-12}\ F/m\times10\times10^{-2}}\vec{a}_R$ $\vec{E} = -539.3\vec{a}_R\ kV/m$

Resposta: Answer: $\vec{E} = -539.3\vec{a}_N$ (kV/m)

$\vec{a}_N$: é o vector unitário perpendicular à recta no sentido desta para o ponto. is the unit vector perpendicular to the straight line and from it to de point.

b) Determine a força, modulo direcção e sentido, a que a carga Q está submetida. Calculate the strength, direction and magnitude which Q is submitted.

$\vec{F} = Q\vec{E}$ $\vec{F} = 1\times10^{-6}\ C(-539.3\times10^3\vec{a}_N)\ V/m$ $\vec{F} = -539.3\vec{a}_N\ mN$

Resposta: Answer: $\vec{F} = -539.3\vec{a}_N$ mN

5) $\vec{E} = \frac{\rho_s}{2\varepsilon}\vec{a}_R$ $\vec{E} = \frac{\rho_l}{2\pi\varepsilon R}\vec{a}_R$ Q = 1 μC d=10cm

a) o campo eléctrico no local de Q provocado pelo plano carregado; the electric field at the place of Q caused by the electrical charged plane;

$\vec{E}_a = \frac{\rho_s}{2\varepsilon}\vec{a}_R$ $\vec{E}_a = \frac{-3\times10^{-6}\ \text{C/m}^2}{2\times8.854\times10^{-12}\ \text{F/m}}\vec{a}_R$

Resposta: Answer: $\vec{E}_a = -169.4\vec{a}_N$ (kV/m)

$\vec{a}_N$: é o vector unitário perpendicular ao plano no sentido deste para o ponto. is the unit vector perpendicular to the plane and from it to de point.

b) o campo eléctrico no local de Q provocado pela recta carregada; the electric field at the place of Q caused by the electrical charged straight line;

$\vec{E}_b = \frac{\rho_l}{2\pi\varepsilon R}\vec{a}_R$ R=d/2 $\vec{E}_b = \frac{-3\times10^{-6}\ \text{C/m}}{2\pi\times8.854\times10^{-12}\ \text{F/m}\times5\times10^{-2}}\vec{a}_R$

$\vec{E}_b = -1079\vec{a}_R$ kV/m

Resposta: Answer: $\vec{E}_b = 1079\vec{a}_N$ (kV/m)

$\vec{a}_N$: é o vector unitário perpendicular à recta no sentido desta para o ponto, que é simétrica a $\vec{a}_R = -\vec{a}_N$. is the unit vector perpendicular to the straight line and from it to de point, that is symmetrical to $\vec{a}_R = -\vec{a}_N$.

c) o campo eléctrico resultante no local de Q; the resultant electric field at the place of Q;

$\vec{E} = \vec{E}_a + \vec{E}_b$ $\vec{E} = -169.4\vec{a}_N + 1079\vec{a}_N$ $\vec{E} = -169.4\vec{a}_N + 1079\vec{a}_N$ $\vec{E} = 909.6\vec{a}_N$ (kV/m)

Resposta: Answer: $\vec{E} = 909.6\vec{a}_N$ (kV/m)

d) a força exercida sobre Q resultante da acção do plano carregado; the force acting on Q resulting from the action of the electrical charged plane;

$\vec{F}_a = Q\vec{E}_a$ $\vec{E}_a = -169.4\vec{a}_N$ (kV/m) $\vec{F}_a = 1\times10^{-6}\ \text{C}\times(-169.4)\vec{a}_R$ kV/m

Resposta: Answer: $\vec{F}_a = -169.4\vec{a}_R$ (mN)

e) a força exercida sobre Q resultante da acção da recta carregada; the force acting on Q resulting from the action of the electrical charged straight line;

$\vec{F}_b = Q\vec{E}_b \qquad \vec{E}_b = 1079\vec{a}_N \quad (\text{kV/m}) \qquad \vec{F}_b = 1\times10^{-6}\ \text{C}\times 1079\vec{a}_N\ \text{kV/m}$

Resposta: Answer: $\vec{F}_b = 1079\vec{a}_N \quad (\text{mN})$

f) de duas formas distintas a força resultante exercida sobre a carga Q. from two distinct forms the resultant force acting on the charge Q.

$\vec{F} = Q(\vec{E}_a + \vec{E}_b) \qquad \vec{F} = 1\times10^{-6}\ \text{C}\, 909.6\ \text{kV/m} \qquad \vec{F} = 909.6\vec{a}_N \quad (\text{m N})$

ou, or

$\vec{F} = \vec{F}_a + \vec{F}_b \qquad \vec{F} = 1079\vec{a}_N - 169.4\vec{a}_N \qquad \vec{F} = 909.6\vec{a}_N \quad (\text{mN})$

Resposta: Answer: como esperado o resultado é o mesmo, $\vec{F} = 909.6\vec{a}_N \quad (\text{m N})$. As expected the result is the same, $\vec{F} = 909.6\vec{a}_N \quad (\text{m N})$.